Scale-Free Density and Current Modulations in a Wedge-Confined Active Gas

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Motivation

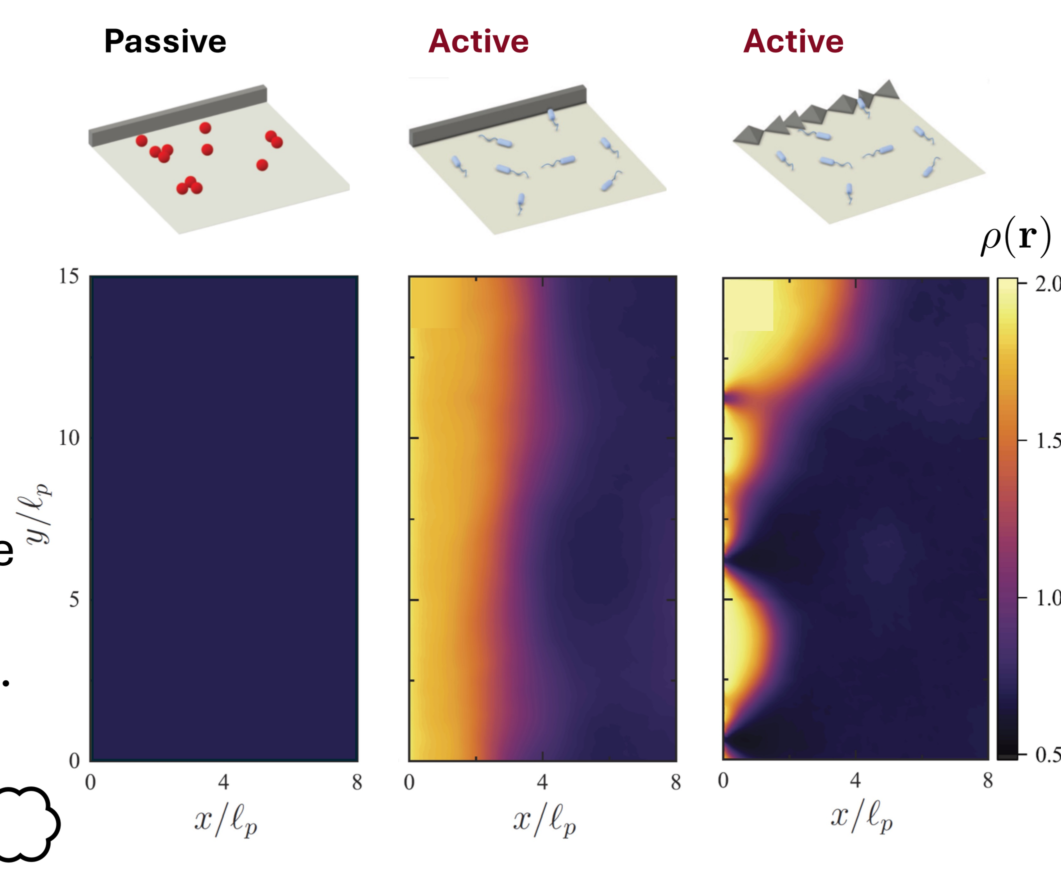
In **active** gases, boundaries induce **density modulations** that extend **into the bulk** and **exert forces** on the boundary in return [1].

These modulations and forces are generically **non-universal**: they depend on the boundary's **own length scales** as well as persistence length ℓ_p of the particles.

A **wedge** introduces no length scale of its own. With ℓ_p the only scale, the modulations and wall pressure reduce to **universal power laws**, fixed by the opening angle β alone.

Such corners have been **realized experimentally** — e.g. shaken granular gases in flower-shaped cells [2].

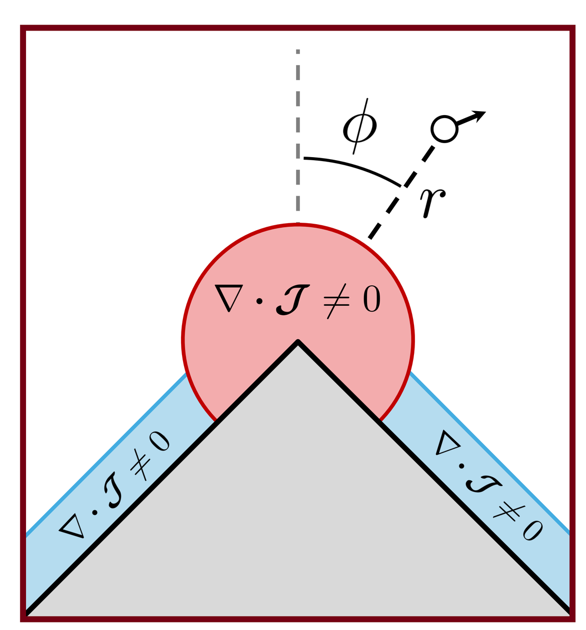
Figure below adapted from [1]



Far-Field Theory

We consider a gas of non-interacting active particles confined to an infinite wedge in two dimensions. In the steady state and away from the boundaries, the density field ρ obeys a Poisson equation with zero-flux boundary conditions,

$$\nabla^2\rho = \frac{1}{D_{\text{eff}}}\nabla\cdot\boldsymbol{\mathcal{J}}, \qquad \partial_\phi\rho\Big|_{\text{wall}} = \frac{r}{D_{\text{eff}}}\mathcal{J}_\phi\Big|_{\text{wall}}.$$

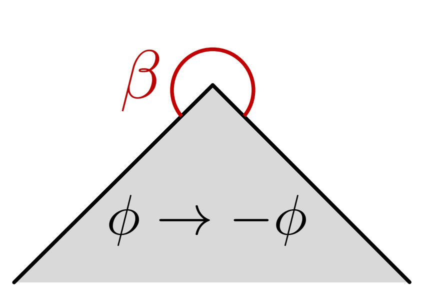


The source term arises from alignment effects from the boundary

$$\boldsymbol{\mathcal{J}} = -\mu\rho\nabla U - \ell_p v\nabla\cdot\left(\mathbf{Q} + D_t\nabla\mathbf{m} - \mu\mathbf{m}\nabla U\right).$$

Through self-consistency arguments, one can show that the dominant source contribution from the infinite boundaries is concentrated in a region near the origin. Therefore, as in electrostatics, one can express the solution to the far-field density as a multipole expansion. The parity of the solution flips in the presence of a symmetry-breaking potential U.

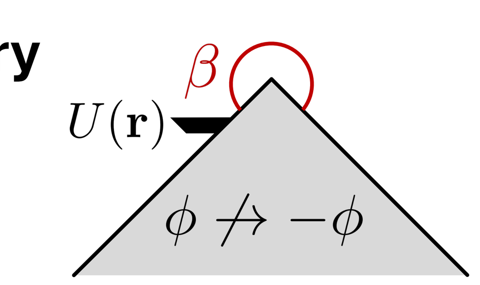
Symmetric Geometry



$$\delta\rho(\mathbf{r}) \sim \frac{A}{r^{\frac{2\pi}{\beta}}}\cos\left(\frac{2\pi\phi}{\beta}\right)$$

$$\mathbf{J}(\mathbf{r}) \propto \frac{A}{r^{\frac{2\pi}{\beta}+1}}\left(\cos\left(\frac{2\pi\phi}{\beta}\right)\hat{\boldsymbol{r}} + \sin\left(\frac{2\pi\phi}{\beta}\right)\hat{\boldsymbol{\phi}}\right)$$

Asymmetric Geometry



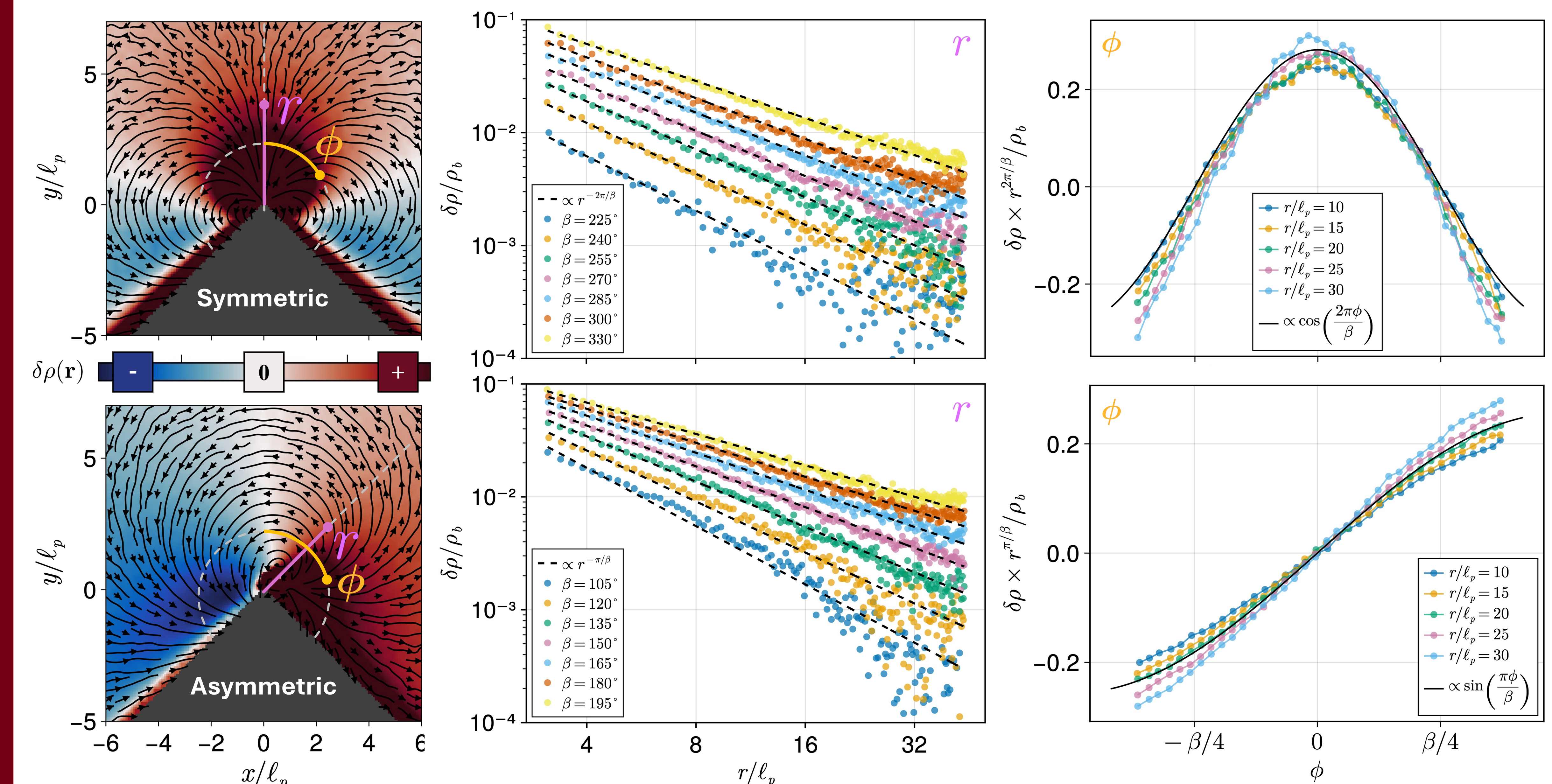
$$\delta\rho(\mathbf{r}) \sim \frac{A}{r^{\frac{\pi}{\beta}}}\sin\left(\frac{\pi\phi}{\beta}\right)$$

$$\mathbf{J}(\mathbf{r}) \propto \frac{A}{r^{\frac{\pi}{\beta}+1}}\left(\sin\left(\frac{\pi\phi}{\beta}\right)\hat{\boldsymbol{r}} - \cos\left(\frac{\pi\phi}{\beta}\right)\hat{\boldsymbol{\phi}}\right)$$

Higher-order corrections correspond to higher-order moments of the Green's function for this geometry:

$$G(\mathbf{r}',\mathbf{r}) = \frac{1}{\beta}\sum_{n\text{ even}}\frac{1}{n}\left(\frac{r'}{r}\right)^{\frac{n\pi}{\beta}}\cos\left(\frac{n\pi\phi'}{\beta}\right)\cos\left(\frac{n\pi\phi}{\beta}\right) + \frac{1}{\beta}\sum_{n\text{ odd}}\frac{1}{n}\left(\frac{r'}{r}\right)^{\frac{n\pi}{\beta}}\sin\left(\frac{n\pi\phi'}{\beta}\right)\sin\left(\frac{n\pi\phi}{\beta}\right)$$

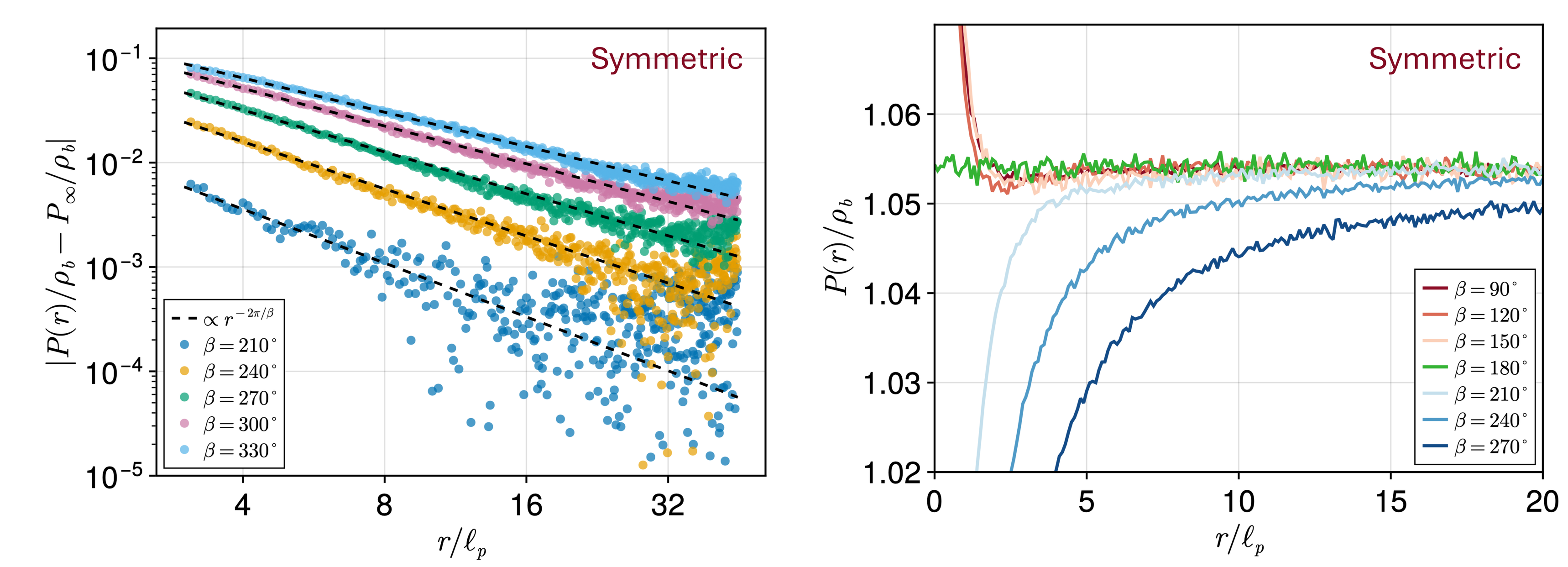
Scale-Free Density Modulations (simulation)



Wall Pressure Variations (simulation)

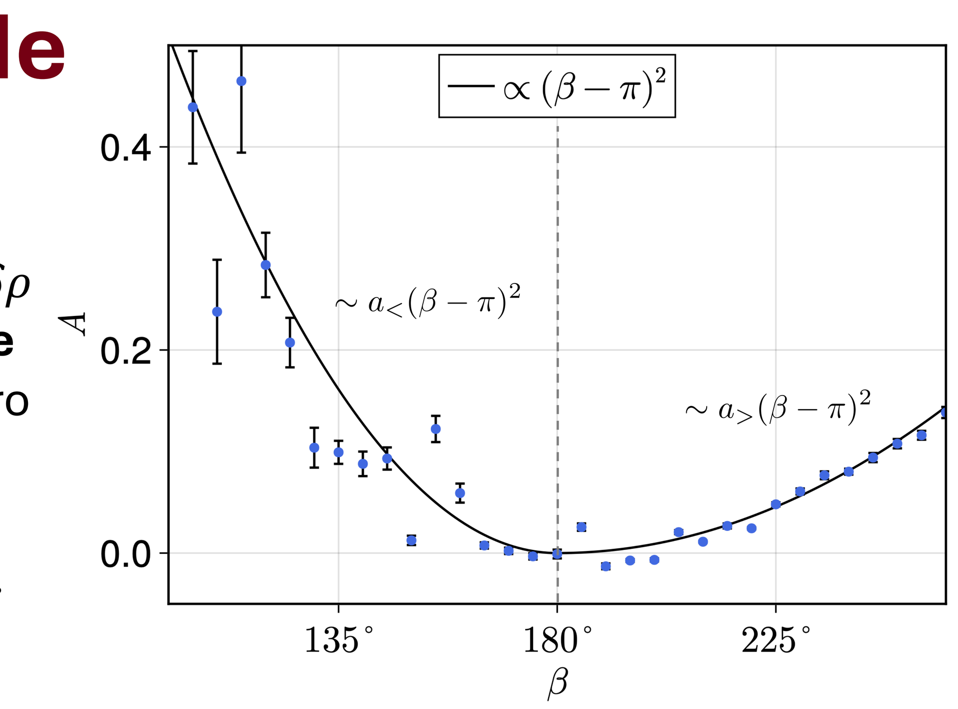
The wall **pressure** depends locally on r and globally on β. It inherits the **scale-free** law $r^{-2\pi/\beta}$ due to **mass conservation**.

For $\beta < \pi$, particles **jam** at the origin, **enhancing** the pressure and washing out the power-law.



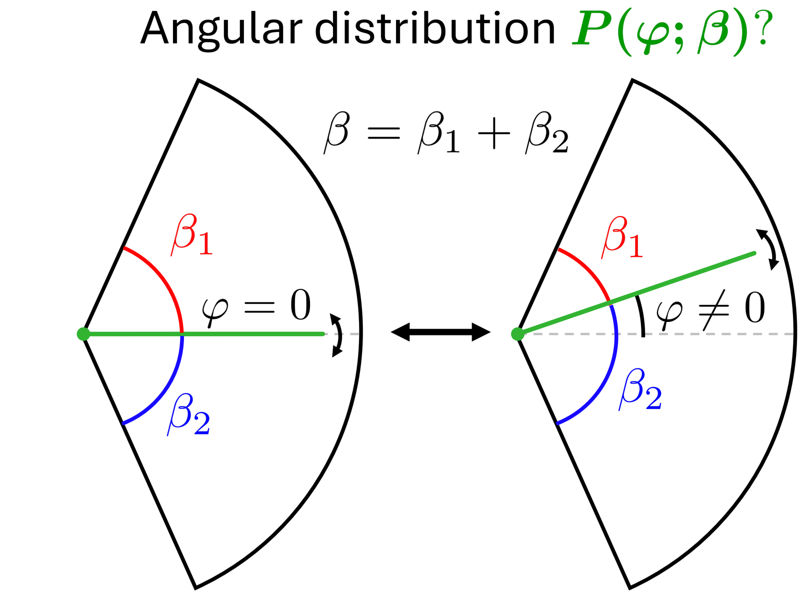
Amplitude (simulation)

The amplitude of $\delta\rho$ stays **sign-definite** across π, so its zero is **quadratic**, steepened by **jamming** for $\beta < \pi$.



Outlook

Angular distribution $P(\varphi;\beta)$?



Ref [1] Ben Dor et al. PRE 105, 044603 (2022)
[2] Deseigne et al. PRL 105, 098001 (2010)